10-96) ① $t(n) = 30{,}000(1.03)^{t-1}$ → $80-55 = 25$ terms

② $t(n) = 60000(1.03)^{t-1}$ $80-65 = 15$ terms

① $\text{Sum} = \dfrac{30{,}000(1.03)^{25} - 30{,}000}{.03} = 1{,}093{,}777$

② $\text{Sum} = \dfrac{60000(1.03)^{15} - 60000}{.03} = 1{,}115{,}934$

The 2nd Plan will pay the most!

~~10-97) $53{,}550 = \dfrac{x(1.08)^1 - x}{.08} + \dfrac{(50{,}000-x)(1.065)^1 - (50{,}000-x)}{.065}$~~

Delete

10-98) a) $8! = 40320$

b) $1 \cdot 7! = 5040$

c) $1 \cdot 7! + 7! \cdot 1 = 10{,}080$

10-99) a) $d = \sqrt{(20-4)^2 + (12-8)^2} = \sqrt{16^2+4^2} = \sqrt{272} = 4\sqrt{17}$

b) $d = \sqrt{(x+3)^2 + (y+5)^2}$

10-100) a) $\dfrac{x}{x+1} = \dfrac{5}{7}$ $7x = 5x + 5$

$2x = 5$

$\boxed{x = 5/2}$

10-100 cont'd b) $\frac{2}{y} = \frac{3}{y+5}$ $2y+10=3y$

$\boxed{10=y}$

c) $\frac{x}{x+1} + \frac{2}{x-1} = \frac{8}{x^2-1}$

$\left(\frac{x-1}{x-1}\right)\left(\frac{x}{x+1}\right) + \left(\frac{x+1}{x+1}\right)\frac{2}{x-1} = \frac{8}{x^2-1}$

$\frac{x^2-x+2x+2}{(x^2-1)} = \frac{8}{x^2-1}$

$\frac{x^2+x+2}{(x^2-1)} = \frac{8}{(x^2-1)}$ $\frac{\cancel{(x^2-1)}(x^2+x+2)}{\cancel{(x^2-1)}} = \frac{8\cancel{(x^2-1)}}{\cancel{(x^2-1)}}$

$x^2+x+2=8$ $x=-3$

$x^2+x-6=0$ $x=2$

$(x+3)(x-2)=0$

d) $\left[\frac{2}{y+5} - \frac{3}{y} = \frac{3}{y+5}\right] y(y+5)$

$2y-3(y+5)=3y$ $\boxed{y=15/4}$

$2y-3y-15=3y$

$-y-15=3y$

$15=4y$

10-101) a) $2x+y=12$ $y=12-2x$ $(x-4)(x-2)=0$

$xy=16$

$x(12-2x)=16$ $x=4$ $y=16/4=4$

$12x-2x^2=16$ $x=2$ $y=16/2=8$

→ solutions are points of intersection

$2x^2-12x+16=0$ $(4,4)$

$x^2-6x+8=0$ $(2,8)$

b) $2x+y=12$ $y=12-2x$

$xy=20$

complex, do not intersect

$x(12-2x)=20$ $\frac{6\pm\sqrt{36-4(1)(10)}}{2} = \frac{6\pm\sqrt{-4}}{2}$

$12x-2x^2=20$

$2x^2-12x+20=0$ $\frac{6\pm 2i}{2} = \boxed{3\pm i = x}$

$x^2-6x+10=0$

$y=12-2(3+i)=6-2i$

$y=12-2(3-i)=6+2i$

10-102) $\tan(20) = .3640$

$\tan(160) = -.3640$

$\tan(200) = +.3640$

$\tan(340) = -.3640$

10-103) a) $\log(x) + \log(x+21) = 2$

$\log x(x+21) = 2$

$10^2 = x^2 + 21x$

$0 = x^2 + 21x - 100$

$(x+25)(x-4)$

$x = 4$

-25 is extraneous

b) $2\log_4(x) - \log_4(3) = 2$

$\log_4 \frac{x^2}{3} = 2$

$4^2 = \frac{x^2}{3}$

$16(3) = x^2$

$48 = x^2$

$\sqrt{48} = x = 4\sqrt{3}$

$-4\sqrt{3}$ is extraneous

c) $\log_2(9x+5) - \log_2(x^2-1) = 2$

$\log_2 \frac{9x+5}{x^2-1} = 2$

$2^2 = \frac{9x+5}{x^2-1}$

$4(x^2-1) = 9x+5$

$4x^2 - 4 = 9x + 5$

$4x^2 - 9x - 9 = 0$

$ac = 36$

$-12 \quad 3$

$4x^2 - 12x + 3x - 9 = 0$

$4x(x-3) + 3(x-3) = 0$

$(4x+3)(x-3)$

extraneous

$x = 3$

d) $\log_7(x+1) + \log_7(x-5) = 1$

$\log_7 (x+1)(x-5) = 1$

$7^1 = x^2 - 5x + x - 5$

$x^2 - 4x - 12$

$(x-6)(x+4)$

$x = 6$

$x = -4$ is extraneous

KEY

Answers

1. 3,628,800
2. 604,800
3. 38,955,840
4. 88
5. 5,112
6. 43,680
7. 56
8. 1
9. 362,880
10. 495
11. 1
12. 1
13. $5! = 120$
14. $2(4!) = 48$
15. $(26)(26)(26)(10)(10)(10) = 17{,}576{,}000$
16. ${}_{25}C_3 = 2300$
17. ${}_{25}P_3 = 13{,}800$ (On a cone, order matters!)
18. ${}_{52}C_5 = 2{,}598{,}960$
19. This is tricky and tough! There are 13 different "types" of cards: twos, threes, fours, ..., Jacks, Queens, Kings, and Aces. We need to choose which of the 13 we want three of (${}_{13}C_1$). Once we choose what type (for example, we pick Jacks) then we need to choose which three out of the four to take (${}_{4}C_3$). Then from the remaining 12 types, we choose which type to have two of (${}_{12}C_1$). Then again we need to choose which two out of the four (${}_{4}C_2$). This gives us $({}_{13}C_1)\cdot({}_{4}C_3)\cdot({}_{12}C_1)\cdot({}_{4}C_2) = 3{,}744$.
20. We already calculated the numbers we need in problems 18 and 19 so: $\frac{3{,}744}{2{,}598{,}960} \approx 0.0014$.
21. Each time we reach in and pull out 8 marbles, order does not matter. The number of ways to do this is ${}_{36}C_8$. This is the number in the sample space, i.e., the denominator. How many ways can we pull out all blue? ${}_{12}C_8$. Therefore the probability is $\frac{{}_{12}C_8}{{}_{36}C_8} \approx 0.0000164$.
22. Same denominator. Now we want to choose 4 from the 12 blue, ${}_{12}C_4$, and 4 from the 4 whites, ${}_{4}C_4$. $\frac{{}_{12}C_4 \cdot {}_{4}C_4}{{}_{36}C_8} \approx 0.0000164$, the same answer!
23. Seven green: ${}_{7}C_7$, one yellow: ${}_{5}C_1$. $\frac{{}_{7}C_7 \cdot {}_{5}C_1}{{}_{36}C_8} \approx 0.0000001652$
24. Here we have to get at least one red: ${}_{8}C_1$, and at least two yellow: ${}_{5}C_2$, but the other five marbles can come from the rest of the pot: ${}_{33}C_5$. Therefore, $\frac{{}_{8}C_1 \cdot {}_{5}C_2 \cdot {}_{33}C_5}{{}_{36}C_8} \approx 0.627$.
25. To get no blue marbles means we want all eight from the other 24 non-blue marbles. $\frac{{}_{24}C_8}{{}_{36}C_8} \approx 0.0243$.