Alg 2 Spring Review

Factor

① $a^2 - 3ab - 130b^2$ = $(a-13b)(a+10b)$

② $x^4 - 13x^2 + 36$ = $(x^2-4)(x^2-9)$ = $(x-2)(x+2)(x-3)(x+3)$

③ $6x^2 - 33x + 45$ = $3(2x^2 - 11x + 9)$ = $3(x-3)(2x-5)$

Solve:

④ $12x^2 + 11x - 15 = 0$ (Factor & use Zero Product Property)

$(3x+5)(4x-3) = 0$ $3x+5=0$ $4x-3=0$

$x = -5/3$ $x = 3/4$

⑤ $12x^2 - 36x + 15 = 0$

$3(4x^2 - 9x + 5) = 0$

$3(2x-5)(2x-1) = 0$

$x = 5/2$ $x = 1/2$

⑥ $2x^2 - 11x + 15 = 0$

$(x-3)(2x-5) = 0$

$x = 3$ $x = 5/2$

⑦ $s^2 + 10s = 0$

$s(s+10) = 0$

$s = 0$ $s = -10$

Rewrite in Vertex Form (complete the square)

(8) $y = x^2 + 4x + 1$

$\{y = ax^2+bx+c\}$
take b → divide by 2 & square result
add this inside ( ) & subtract it outside ( )

$y = (x^2 + 4x + \underline{4}) - (\underline{4}) + 1$

$(\frac{4}{2})^2 = 4$

$\boxed{y = (x+2)^2 - 3}$

$\boxed{\text{vertex } (-2, -3)}$

(9) $y = 2x^2 - 8x + 1$

$y = 2(x^2 - 4x + \underline{\ \ }) + 1 - 4(2)$ factor out "a" then complete the square

$y = 2(x^2 - 4x + \underline{4}) + 1 - 8$

$b = -4$

$(\frac{b}{2})^2 = (\frac{-4}{2})^2 = 4$

$\boxed{y = 2(x-2)^2 - 7}$

$\boxed{\text{vertex } (2, -7)}$

Solve by complete the square

(10) $x^2 + 6x - 3 = 0$

$b = 6 \quad (\frac{b}{2})^2 = (\frac{6}{2})^2 = 9$

$x^2 + 6x = 3$

$x^2 + 6x + 9 = 3 + 9$

$(x+3)^2 = 12$

$\sqrt{(x+3)^2} = \pm\sqrt{12}$

$x + 3 = \pm 2\sqrt{3} \rightarrow \boxed{x = -3 \pm 2\sqrt{3}}$

Solve by completing the square

(11) $x^2 - 12x + 7 = 0$ $\quad \left(\frac{b}{2}\right)^2 =$

$x^2 - 12x = -7$ $\quad \left(\frac{-12}{2}\right)^2 = 36$

$x^2 - 12 + 36 = -7 + 36$

$(x-6)^2 = 29$

$\sqrt{(x-6)^2} = \sqrt{29}$

$x - 6 = \pm\sqrt{29}$

$\boxed{x = 6 \pm \sqrt{29}}$

(12) $x^2 + 8x = 11$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = 16$

$x^2 + 8x + 16 = 11 + 16$

$(x+4)^2 = 27$

$\sqrt{(x+4)^2} = \pm\sqrt{27}$

$x + 4 = \pm 3\sqrt{3}$

$\boxed{x = -4 \pm 3\sqrt{3}}$

Divide.

(13) $(9x^5 - 3x^3 + 27x) \div 3x$ ← one term use monomial division

$$\frac{9x^5 - 3x^3 + 27x}{3x} = \boxed{3x^4 - x^2 + 9}$$

(14) $y^5 + 6y^3 - y^2 + 9y - 3 \div (y^2 + 3)$ long division

$$\begin{array}{r} y^3 + 3y - 1 \\ y^2+3 \overline{)\, y^5 + 6y^3 - y^2 + 9y - 3} \\ \text{change signs } \underline{-(y^5 + 3y^3)} \\ 0 + 3y^3 - y^2 + 9y - 3 \\ \text{change signs } \underline{-(3y^3 + 9y)} \\ 0 - y^2 + 0 - 3 \\ \underline{+y^2 + 3} \\ 0 \end{array}$$

$\boxed{y^3 + 3y - 1}$

also

	y^3	$+3y$	-1
y^2	y^5	$3y^3$	$-y^2$	0
$+3$	$3y^3$	0	$9y$	-3

polyclaken

(15) Divide

$$3y^3 - 2y^2 + y - 1 + \frac{1}{3y-1}$$

$5y-1 \;\big|\; 9y^4 - 9y^3 + 5y^2 - 4y + 2$

Remainder 1

change signs → $-9y^4 + 3y^3$

$-6y^3 + 5y^2$

change signs → $+6y^3 - 2y^2$

$3y^2 - 4y$

change signs → $-3y^2 + y$

$-3y + 2$

$+3y - 1$

1

also

	$3y^3$	$-2y^2$	$+y$	-1	← remainder
$3y$	$9y^4$	$-6y^3$	$3y^2$	$-3y$	1
-1	$-3y^3$	$+2y^2$	$-y$	1	

Polynomial

(16)

$$x^2 - 4x + \frac{4}{x+3}$$

$x+3 \;\big|\; x^3 - x^2 - 12x + 4$

$-x^3 \mp 3x^2$

$-4x^2 - 12x$

$+4x^2 + 12x$ ↓

$0 \quad 0 \quad +4$ remainder

also

synthetic division

$-3 \;\big|\; 1 \quad -1 \quad -12 \quad 4$

$\qquad\quad -3 \quad 12$

$1 \quad -4 \quad 0 \;\big|\; 4$

$x^2 - 4x$ + remainder 4

(16) $x^2 - 4x$ + remainder 4

	x^3	$-4x^2$	0	4
x				
$+3$	$3x^3$	$-12x$		

(Polydivide)

(17) $4x^3 + 3x^2 + 2x - 18 + \frac{1}{x-3}$

$x-3 \overline{)\ 4x^4 - 9x^3 - 7x^2 - 6x + 1}$

change signs

$-4x^4 + 12x^3$

$3x^3 - 7x^2$

$-3x^3 + 9x^2$

$2x^2 - 6x$

$+2x^2 - 6x$

$+1$ ← remainder

Synthetic div

$3 \mid 4 \quad -9 \quad -7 \quad -6 \quad 1$

$\quad\quad 12 \quad 9 \quad 6 \quad 0$

$4 \quad 3 \quad 2 \quad 0 \mid 1$ remainder 1

$4x^3 + 3x^2 + 2x + \frac{1}{x-3}$ ← remainder 1

Polydivide

	$4x^4$	$3x^3$	$2x^2$	0	1
x					
-3	$-12x^3$	$-9x^2$	$-6x$	0	

(18) Solve

$\frac{2}{15} + \frac{1}{3} = \frac{x}{15}$ clear fractions times by 15

$15\left(\frac{2}{15}\right) + 15^{5}\left(\frac{1}{3}\right) = 15\left(\frac{x}{15}\right)$

$2 + 5 = x$

$7 = x$

(19) $\frac{4}{3x+5} = \frac{3}{2x}$

$8x = 9x - 15$

$+15 \quad -8x \quad -8x \quad +15$

$15 = x$

(20) $8x^2 - 2x - 3 = 0$ Factor to Solve & use Zero Product Property

$(2x+1)(4x-3) = 0$

$2x + 1 = 0$ $\quad 4x - 3 = 0$

$x = -1/2$ $\quad x = 3/4$

(21) $p(3p+2) = 5 \rightarrow 3p^2 + 2p - 5 = 0$

$(p-1)(3p+5) = 0$

$p = 1 \quad p = -5/3$

(22) Solve

$t^2 + 4t = 21$

$t^2 + 4t - 21 = 0$

$(t+7)(t-3) = 0$

$t = -7 \quad t = 3$

(23) $|2x-1| < 5$

Write 2 inequalities less than

$2x - 1 < 5$

$2x < 6$

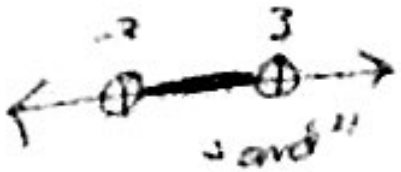
$x < 3$

$2x - 1 > -5$

$2x > -4$

$x > -2$

case II flip the equality & change the sign



$-2 < x < 3$

(24) $3|x+2| - 1 \leq 8$

$+1 \quad +1$ (add 1 to both sides)

$\frac{3}{3}|x+2| \leq \frac{9}{3}$ (divide both sides by 3)

$|x+2| \leq 3$

split into 2 inequalities

case II flip inequality & change sign

$x+2 \leq 3$

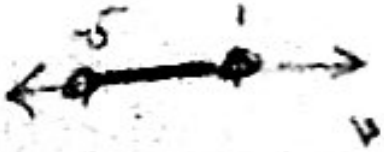
$-2 \quad -2$

$x \leq 1$

$x+2 \geq -3$

$-2 \quad -2$

$x \geq -5$



$-5 \leq x \leq 1$

Solve

(25) $V = hw^2$ for "w"

divide by h: $\frac{V}{h} = \frac{hw^2}{h}$

root both sides: $\sqrt{\frac{V}{h}} = \sqrt{w^2}$

$$\boxed{w = \sqrt{\frac{V}{h}}} \quad \left(\text{also } \frac{\sqrt{Vh}}{h}\right)$$

↑ rationalized

(26) $F = \frac{kq_1q_2}{r^2}$ for "r"

multiply by r^2: $Fr^2 = kq_1q_2$

divide by F: $\frac{Fr^2}{F} = \frac{kq_1q_2}{F}$

root both sides: $\sqrt{r^2} = \sqrt{\frac{kq_1q_2}{F}}$

$$\boxed{r = \sqrt{\frac{kq_1q_2}{F}}}$$

$F(r^2) = \frac{kq_1q_2}{r^2}(r^2)$

(27) $\sqrt{\frac{F}{k}} = x$ for "F"

square both sides: $\left(\sqrt{\frac{F}{k}}\right)^2 = x^2$

multiply by k: $\frac{F}{k}(k) = x^2(k)$

$$\boxed{F = kx^2}$$

Simplify: Multiply

$i = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

(29) i^{34} divide $34/4$ = 8 remainder 2

$= (i^4)^8 (i^2) \longrightarrow (-1)$

$(1)^8 \cdot i^2 \rightarrow (1)(1)(1)(1)(1)(1)(1)(1)(1)i^2 = \boxed{-1}$

(30) i^{63} divide $63/4$ = 15 remainder 3

$(i^4)^{15} \cdot i^3$

goes to $(1)^{15} \cdot -i = \boxed{-i}$

(31) $(x+5)^3 \rightarrow \binom{3}{0}x^3 + \binom{3}{1}x^2 \cdot 5 + \binom{3}{2}x \cdot 5^2 + \binom{3}{3}5^3$

$\boxed{x^3 + 15x^2 + 75x + 125}$

(32) $(3x+5)(9x-2)$ FOIL

$27x^2 - 6x + 45x - 10 = \boxed{27x^2 + 39x - 10}$

(33) $f(x) = x^2 + 2x - 15$ Find x-intercepts

← $y = 0$ at x-intercepts

$0 = x^2 + 2x - 15$

$0 = (x+5)(x-3)$ $x = -5$ $x = 3$

$(-5, 0)$ $(3, 0)$ { x-intercepts

(34) Find x-intercepts

$f(x) = 9x^2 + 12x + 4$

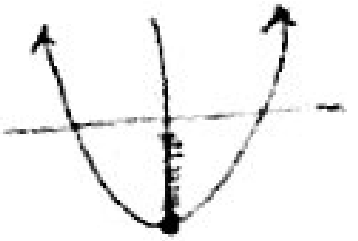
$0 = 9x^2 + 12x + 4$

$0 = (3x+2)(3x+2)$ $\quad x = -2/3$

2 x-intercepts at $(-2/3, 0)$

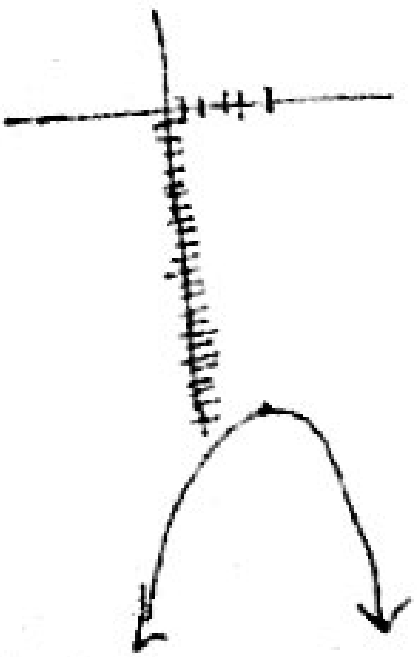
(35) Graph + describe transition

$f(x) = x^2 - 9$ parent graph $y = x^2$



$y = x^2 - 9$
translated down 9 units

(36) $f(x) = -(x-4)^2 - 25$ parent graph $y = x^2$



graph is flipped across the
x-axis
& shifted 4 units to the right
& shifted 25 units down

(34) Find x-intercepts

$f(x) = 9x^2 + 12x + 4$

$0 = 9x^2 + 12x + 4$

$0 = (3x+2)(3x+2)$ $\quad x = -2/3$

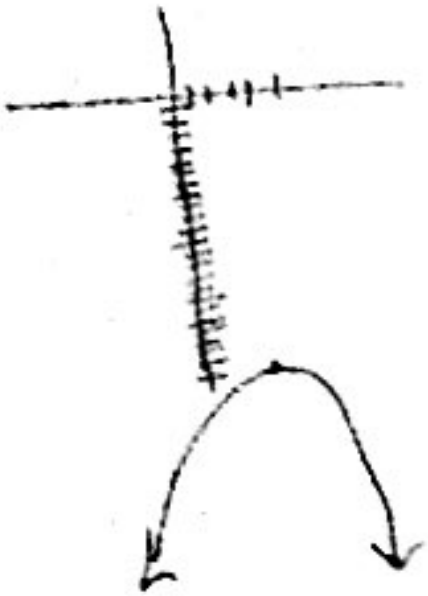
2 x-intercepts at $(-2/3, 0)$

(35) Graph + describe transition

$f(x) = x^2 - 9$ parent graph $y = x^2$

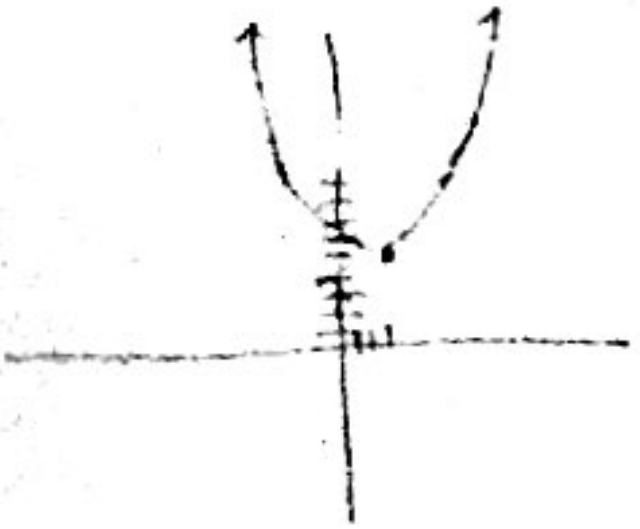
$y = x^2 - 9$ translated down 9 units

(36) $f(x) = -(x-4)^2 - 25$ parent graph $y = x^2$



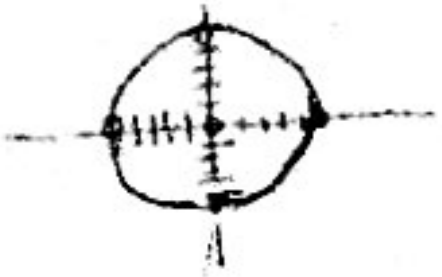
graph is flipped across the
- x-axis
- & shifted 4 units to the right
- & shifted 25 units down

(37) $f(x) = 2(x-3)^2+5$



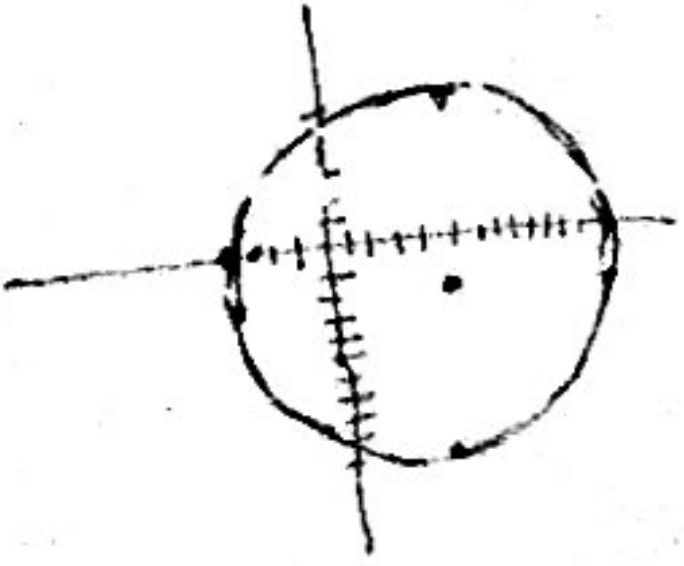
f(x) is stretched vertically by a factor of 2
it is shifted to the right 3 units
& it is shifted up 5 units

(38) $x^2+y^2 = 25$ ← eqn of a circle of radius 5



center of this circle is (0,0)

(39) $(x-5)^2+(y+2)^2 = 64$ circle of radius 8
center is at (5, -2)



(40) Solve for the inverse & decide if it is a function

$f(x) = \frac{1}{x-1}$

$y = \frac{1}{x-1}$ <switch x & y>

$(y-1)x = \frac{1}{y-1}(y-1)$ <multiply by "y-1">

$\frac{(y-1)(x)}{x} = \frac{1}{x}$ <divide by x>

$y-1 = \frac{1}{x}$

$y = 1 + \frac{1}{x}$ yes $f^{-1}(x) = 1 + \frac{1}{x}$ is a function

(41) $f(x) = (1-2x)^2 + 5 \rightarrow y = (1-2x)^2 + 5$

switch x & y.

$\frac{x}{-5} = \frac{(1-2y)^2}{-5} + \frac{5}{-5}$ <not both sides>

$x-5 = (1-2y)^2 \rightarrow \sqrt{x-5} = \sqrt{(1-2y)^2}$

$1-2y = \pm\sqrt{x-5}$

$\frac{-2y}{-2} = \frac{-1 \pm \sqrt{x-5}}{-2}$

$y = \frac{1 \pm \sqrt{x-5}}{2}$ ← 2 outputs for every input!

not a function

Simplify

(42) $(3+5i)-(2+6i)$ · $3+5i-2-6i$

$\boxed{1-i}$

(43) $(8+2i)(5-4i)$ Foil

$40+10i-52i-8i^2$ ← simplify $i^2=-1$

$40-22i-(-8)$

$\boxed{48-22i}$

(44) $(7+8i)(7-8i)$ · $49-64i^2$

$49+(-1)(-64)$

$49+64$ · $\boxed{113}$

Simplify

$f(x)=3x-5$

$g(x)=x^2+1$

$h(x)=\frac{1}{x}$

(45) $f(g(-2))$ · $f((-2)^2+1)$ · $f(5)=3(5)-5$ · $\boxed{10}$

(46) $g(f(x))$ · $g(3x-5)$ · $(3x-5)^2+1$

$=9x^2-30x+25+1$

$=\underline{9x^2-30x+26}$

(47) $f(h(9))$ · $f\left(\frac{1}{9}\right)$ · $3\left(\frac{1}{9}\right)-5$ · $\frac{1}{3}-5$ · $\frac{1}{3}-\frac{15}{3}$ · $\boxed{\frac{-14}{3}}$

(48) $\begin{cases} x+3y=7 \\ 2x-4y=24 \end{cases}$ ← times by (-2)

$-2x-6y=-14$

$2x-4y=24$

$-10y=10$

$y=-1$

$x+3(-1)=7$

$x=10$

$(10,-1)$

(49) $\begin{cases} y=2x-1 \\ 3x-y=-1 \end{cases}$ use substitution

$3x-(2x-1)=-1$

$3x-2x+1=-1$

$\boxed{x=-2}$

$y=2(-2)-1$

$\boxed{y=-5}$

$(-2,-5)$

(50) Student has $1 & $5 in wallet with 15 bills in all

$\$=47$ how many of each type?

$x=$ $1 bill

$y=$ $5 bill

$-1x+y=15$

$1x+5y=47$

$x-y=-15$

$4y=32$

$y=8$

$\boxed{\begin{array}{l} 8\{\$5 \text{ bills} \\ 7\{\$1 \text{ bills} \end{array}}$

(51) youth group 26 members
5 chaperones drive a car or van
vans seat 7, cars seat 5
31 people

$-5(C + V = 5)$

$7V + 5C = 31$
$-5V - 5C = -25$
$2V + 0 = 6$

$\boxed{\text{Vans} = 3}$ $\boxed{\text{cars} = 2}$

(52) Convert to radians or degrees

$\left(\frac{95^\circ}{1}\right)\left(\frac{\pi}{180^\circ}\right) = \boxed{\frac{19}{36}\pi}$

(53) $\left(-\frac{5\pi}{7}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{900^\circ}{7} = 128.57^\circ$

Find the reference angle

(54) 135° 

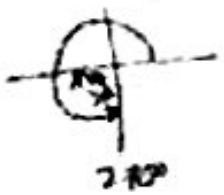
$\theta = 180 - 135 = 45^\circ$

(55) 222 

$\theta = 222 - 180 = 42^\circ$

(56) reference angle

$\theta = 360° - 330° = 30°$

(57) 

$\theta = 270° - 180 = 90°$
also
$360° - 270° = 90°$

What are the trig values for $\sin\theta, \cos\theta, \tan\theta$

(58) for 30° $\sin 30 = \frac{1}{2}$

$\cos 30 = \sqrt{3}/2$

$\tan 30 = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

(59) for 60° $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

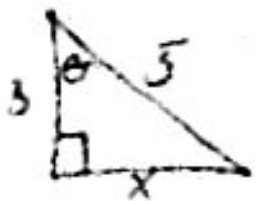
(60) for 0° $\sin 0° = 0$

$\cos 0° = 1$

$\tan 0° = 0$

(61) for 45° $\sin 45° = \frac{\sqrt{2}}{2}$ $\cos 45° = \frac{\sqrt{2}}{2}$ $\tan 45° = 1$

(62)

a) 

$\sin\theta = \frac{x}{5}$

$\cos\theta = \frac{3}{5}$

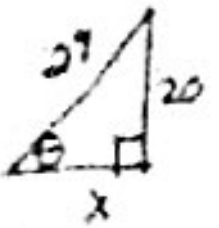
$\tan\theta = \frac{x}{3}$

$x^2 + 3^2 = 5^2$

$x^2 = 25 - 9$

$x^2 = 16$

$\boxed{x = 4}$

b) 

$\sin\theta = \frac{20}{29}$

$\cos\theta = \frac{x}{29}$

$\tan\theta = \frac{20}{x}$

$x^2 + 20^2 = 29^2$

$x^2 + 400 = 841$

$x^2 = 441$

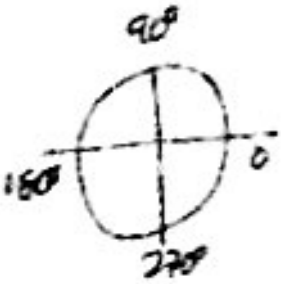
$x = \sqrt{441}$

$\boxed{x = 21}$

(63)

$\boxed{\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$

(64)



	0	90	180	270°
sin	0	1	0	-1
cos	1	0	-1	0
tan	0	und	0	und

(63)

a) $8P3$ $\quad \frac{n!}{(n-r)!}$ $\quad \frac{8!}{5!} = 336$

b) $4C3$ $\quad \frac{n!}{(n-r)!\,r!} = 4$

c) $6P4 = \frac{6!}{2!} = 360$

d) $10C7 = \frac{10!}{3!\,7!} = 120$

(64) Find the common ratio

a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ $\quad \boxed{r = \frac{1}{2}}$

b) $32, 8, 2, \frac{1}{2}$ $\quad \boxed{r = \frac{1}{4}}$

c) $6, 18, 54, 112$ $\quad \boxed{r = 3}$

(67) Choose 10 mangos from a box of 50

$50C10$ or $\binom{50}{10}$

(68) $y = ab^{-x}$

(69) A radian is the central arc subtended by the radius of a circle

(70) Investigate

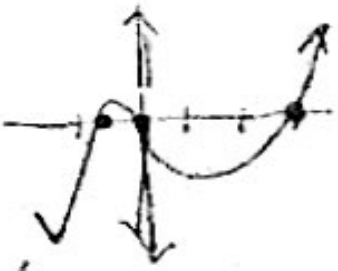
a) $y = 2x^3 - 5x^2 - 3x = x(2x^2 - 5x - 3)$

$= x(2x+1)(x-3)$

the polynomial is a cubic with

zeros at $0, -\frac{1}{2}, 3$

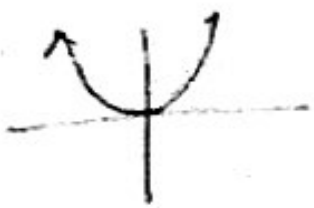
and a orientation that rises as you move to the right



range & domain $(-\infty, \infty)$

b) $y = 3x^4 - 6x^3 + 12x^2$

$y = 3x^2(x^2 - 2x + 4)$



Zero's at $(0,0)$

minimum of $y = 0$ at $x = 0$

domain $(-\infty, \infty)$

range $[0, \infty)$

(71)

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ annual interest $n = 1$

$A = 1000\left(1 + .05\right)^{6}$

$= 1000(1.05)^{6} = \boxed{1340.10}$

(72) Solve $\begin{cases} x - y + z = -1 & (1) \\ x + y + 3z = -3 & (2) \\ 2x - y + 2z = 0 & (3) \end{cases}$

$x - y + z = -1$ (1)
$x + y + 3z = -3$ (2)
$2x + 4z = -2$

$-(x - y + z = -1)$ (1) $\rightarrow$ $-x + y - z = 1$
$2x - y + 2z = 0$ (3) $\quad 2x - y + 2z = 0$

$-2(x + z = 1) \rightarrow -2x - 2z = -2$

$2x + 4z = -2$
$-2x - 2z = -2$
$2z = -4$
$z = -2$

$3 - y - 2 = -1$
$1 - y = -1$
$-y = -2$
$y = 2$

$2x + 4(-2) = -2$
$2x - 8 = -2$
$2x = 6 \quad x = 3$

$\boxed{(3, 2, -2)}$

(73) Eqn of parabola through $(-2,-32)(0,-10)(2,-12)$

$y = ax^2 + bx + c$

$-32 = 4a - 2b + c$ $\longrightarrow$ $-32 = 4a - 2b - 10$

$+10 \quad +10$

let $c = -10$

$-22 = 4a - 2b$

$-10 = c$

$-12 = 4a + 2b + c$ $\longrightarrow$ $-12 = 4a + 2b - 10$

$+10 \quad +10$

$-2 = 4a + 2b$

$-22 = 4a - 2b$

$\frac{-24}{8} = \frac{8a}{8}$

$a = -3$

$-2 = 4(-3) + 2b$

$-2 = -12 + 2b$

$10 = 2b$

$b = 5$

$\boxed{y = -3x^2 + 5x - 10}$